# WAVE INTERACTION IN DOUBLY PERIODIC STRUCTURES

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## Summary

A rigorous analysis is presented for the propagation in a doubly periodic structure. General wave characteristics are established and various types of wave interaction are identified. Numerical examples are shown to illustrate new interesting physical phenomena.

## Introduction

In this paper, we present a rigorous analysis of a class of doubly periodic structures which are composed of uniform dielectric layers and singly periodic layers of two different periods. Such a type of structures have been suggested for realizing the coupling of three modes for enhancing the Bragg reflection for the design of filters. However, the guiding characteristics of doubly periodic structures have been analyzed only approximately by perturbation methods [1,2]. While approximate methods are desirable for constructing simple analytical results that are useful for practical design considerations, they often require a priori knowledge of the basic physical processes involved. So far, the physical picture of wave phenomena in multiply periodic structures has not been well developed, and a better understanding of basic wave processes involved is needed for establishing a sound basis on which desired simple approximations can be properly developed. We have observed that with grating of rectangular profile, the class of multilayer structures can be formulated exactly as an electromagnetic boundary-value problem. The purposes of such an exact analysis are threefold:

(1) To provide a basis for the understanding of global characteristics of wave propagation in a doubly periodic structure.

(2) To identify various types of wave interaction that may exist in doubly periodic structures, but not in singly periodic ones.

(3) To give accurate numerical data against which approximate results can be judged. In adition, our approach is to obtain accurate numerical results for doubly periodic structures (DPS), and compare them to those of well known singly periodic structures (SPS), thereby clearly identifying the unique properties of DPS.

## Boundary-Value Problem of DPS

Consider a dielectric layer that is periodically corrugated on both boundary surfaces with two different periods, as shown in Fig.1. Both periodic layers can be regarded as dielectric gratings consisting of uniform rectangular rods placed periodically in the x direction. The period and thickness of the upper grating (grating A) are a and $t_a$, and those of the lower grating (grating B) are b and $t_b$, respectively. The corrugated structure has a dielectric constant $_f$ and the central portion, which can be taken as a uniform layer (film), has a thickness $t_f$. Such a corrugated structure is generally sandwiched between two half-space dielectic media. For simplicity, the upper and lower half-spaces will be referred to as the air and substrate regions, and their dielectric constants are denoted by $_a$ and $_s$, respectively. Although Fig.1 shows the special case of rectangular corrugations of the dielectric film, the theory to be presented below holds for dielectric gratings of arbitrary profile, as shown for the case of SPS [3,4].

The structure under consideration contains two physically separated gratings of different periods. Furthermore, the guiding of waves along the DPS may be viewed as a process of multiple reflections by the two gratings. The reflection matrix of a single grating sandwiched between two uniform half spaces of different dielectric constants had been



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previously defined [3-5]. Using the reflection matrix of a grating as a building block, an exact formulation of the boundary-value problem of DPS had been carried out. Both analytical and numerical results had obtained on the basis of the exact formulation, as described below.

## General Guiding Characteristics of DPS

By the exact formulation, we have defined a dispersion relation for the class of DPS under consideration. Some general properties of the dispersion roots are then obtained, thereby establishing the global guiding characteristics of the Brillouin diagram. We summarize here these general properties along with their implications to the design of DPS for applications as either filters or antennas.

### A. Commensurate and noncommensurate periods

So far, the two periods of the DPS are assumed to be arbitrary; their ratio can be a rational or irrational number and the theory described above holds for both cases. However, it should be noted that in a numerical analysis using a computer, an irrational number is always approximated by a rational one with a finite number of digits. Therefore, we can obtain numerical results only for DPS of commensurate periods.

In the commensurate case, the ratio of the periods can be expressed in the general form:

$$\frac{a}{b} = \frac{m}{N} \qquad (1)$$

where M and N are two integers without any common factor. It then follows from the last equation that the least common multiple of a and b is

$$d = Na = Mb \qquad (2)$$

which means physically that a DPS with commensurate periods can be regarded as a SPS with the period d, as determined by (1) and (2). In other words, a DPS with commensurate periods must exhibit the propagation characteristics of SPS, which are considered to be well understood [6].

On the other hand, in the noncommensurate case, the ratio of the two periods is an irrational number, which contains an infinite number of digits. In view of (1), a DPS of noncommensurate periods can be taken as a limiting case of commensurate ones, with the integers M and N approaching infinity. Thus, the period d in (2) then becomes infinite. A physical consequence of such a limiting process is that as the number of digits is increased indefinitely, the bound wave-wave region ($0 < k_o < \pi/d$) is continually reduced to zero. Thus, a DPS of noncommensurate periods is always leaky as a waveguide, at least, in principle. The magnitude of the leakage depends also on other structure parameters, and it has to be determined by the eigenvalue of the boundary-value problem formulated in the preceding section. It should be pointed out that the introduction of the second grating into the waveguide was originally intended for enhancing the Bragg reflection in the bound-wave region for the filter design [1,2]. In view of the reduced bound-wave region, the design of DPS for filter applications may be frustrated by the leakage of energy which may degrade the performance of the device. Furthermore, the reduction of the bound-wave region may be interpretted to be due to the radiation of multiple harmonics. Such multiple-beam radiations may have far-reaching implications for the design of antennas.

### B. General characteristics of dispersion root

From the exact dispersion relation, we have proved mathematically the following three properties concerning the dispersion roots of DPS:

(D1) If $k_{xoo}$ is a dispersion root, so is $-k_{xoo}$.

(D2) If $k_{xoo}$ is a dispersion root, so is $k_{xpq}$, for any integers p and q.

(D3) If $k_{xoo} = p\pi/a + q\pi/b - j\alpha$ is a dispersion root, so are $k^*_{xoo}$, $k_{x,-p,-q}$, and $k^*_{x,-p,-q}$.

The first property (D1) states that the guidiance of waves along the structure is reversive, even though the structure may not possess a reflection symmetry in the x-direction. Such a property should have been expected, because the structure is reciprocal. The second property (D2) states the dispersion roots possess the translation symmetry with the periods of the gratings, even though the structure itself does not. The last property (D3) states that under the phase matching condition, the space harmonics exist in pair. Such a property will be very important to the determination of the possible types of mode coupling in a DPS, as we shall show next.

## Numerical Results

For the structure shown in Fig.1 and with the period a of grating A as a unit, we choose the following set of parameters: b=1.1a, for the period of grating B; $t_a=t_b=0.4a$, for the thicknesses of gratings A and B; $t_f=1.8a$, for the thickness of the film. In addition, the refractive indices of the materials are chosen to be: $n_a=n_s=1.0$ and $n_f=1.9$. For such a structure, we have examined the propagation characteristics in general, but focus our attention on the stopband regions which are of primary interest for filter designs.

For the design of DPS, it is desirable to determine first the unperturbed dispersion curves by considering a uniform dielectric waveguide as an approximation for the DPS. Here, we take the average-thickness approximation by replacing each of the two periodic layers by a uniform one with a half of the original thicknesses. The dispersin curves of the lowest two TE modes are shown in Fig.2, with the labels (1,0,0) and (2,0,0). Each one of these is regarded as the fundamental harmonic of the mode. In the presence of grating A, other harmonics with a phase shift of a multiple of $2\pi/a$ are excited; the dispersion curves for the m=-1 space harmonics for the two lowest modes are shown with the labels (1,-1,0) and (2,-1,0). Similarly, the dispersion curves for n=-1 space harmonics due to grating B are shifted by $2\pi/b$ and are labelled by (1,0,-1) and (2,0,-1). Among the three integers in each labelling, the first is the mode index, the second and the third are the harmonic indices due to gratings A and B, respectively. It is well known from the theory of mode coupling that the dispersion curves of a perturbed structure will follow closely those of the unperturbed one, except in the vicinity of their intersection points. Near the intersection points, the space harmonics are strongly coupled, resulting in complex dispersion roots that represent decaying waves due to total reflections, known as the Bragg Phenomenon.

It is noted that the structure parameters for the present analysis were chosen on the basis of these unperturbed dispersion curves. They were chosen such that the intersection of three unperturbed dispersion curves at the same point can occur, as shown in the boxed area marked by A in Fig.2. This means that in the case of DPS, three-mode coupling, which cannot be achieved by singly periodic structures, can now be realized. The three-mode coupling mechanism had been suggested as a mean for the enhancement of mode coupling [1,2]. We have carefully investigated this potential three-mode coupling case, and the detailed stopband structures are shown as insets in Fig.2. In Inset 1, we examine the two limiting cases, $t_a=0$ and $t_b=0$, corresponding to two SPS where only two-mode couplings can take place. As shown, we have two usual stopbands for the two limiting cases. Inset 2 shows the stopbands of the DPS. Evidently, in the presence of the two gratings, all the modes are strongly coupled, so that the two stopbands occur at the same phase constant, as expected. Comparing the two insets, we observe that, contrary to expectation, one stopband is reduced in the case of the DPS, while the other stay practically unchanged. More importantly, in the case of DPS, there exists in the region of overlap between the two stopbands. Since complex roots must occur in pairs, this means that there exist four roots, with one extra or unexpected root, for the anticipated three-mode coupling case. We have examined the overall space harmonics on the basis of the mathematical properties stated in the preceding section. For the present case, it is actually an interaction of four modes, which are automatically accounted for in our exact dispersion relation. More numerical data have been obtained, showing many interseting physical phenomena. Their implications to the design of DPS for applications as filters or antennas will be systematically illustrated in the presentation.

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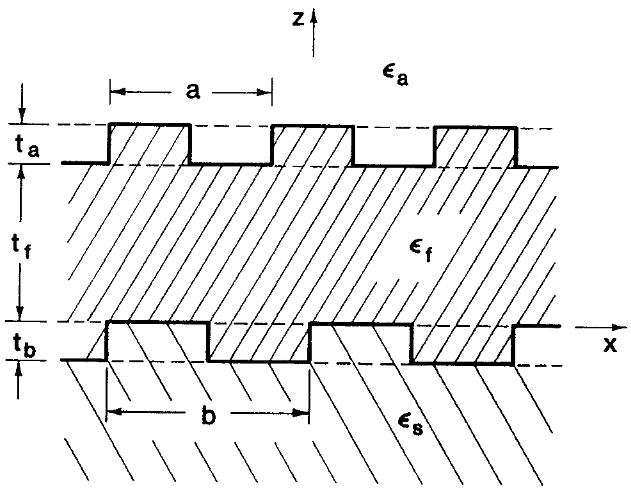


Fig. 1. Configuration of doubly periodic structure

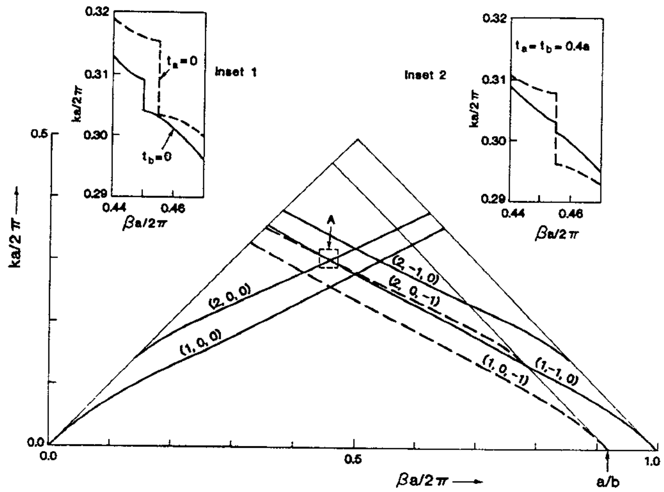


Fig. 2. Dispersion curves of doubly periodic structure